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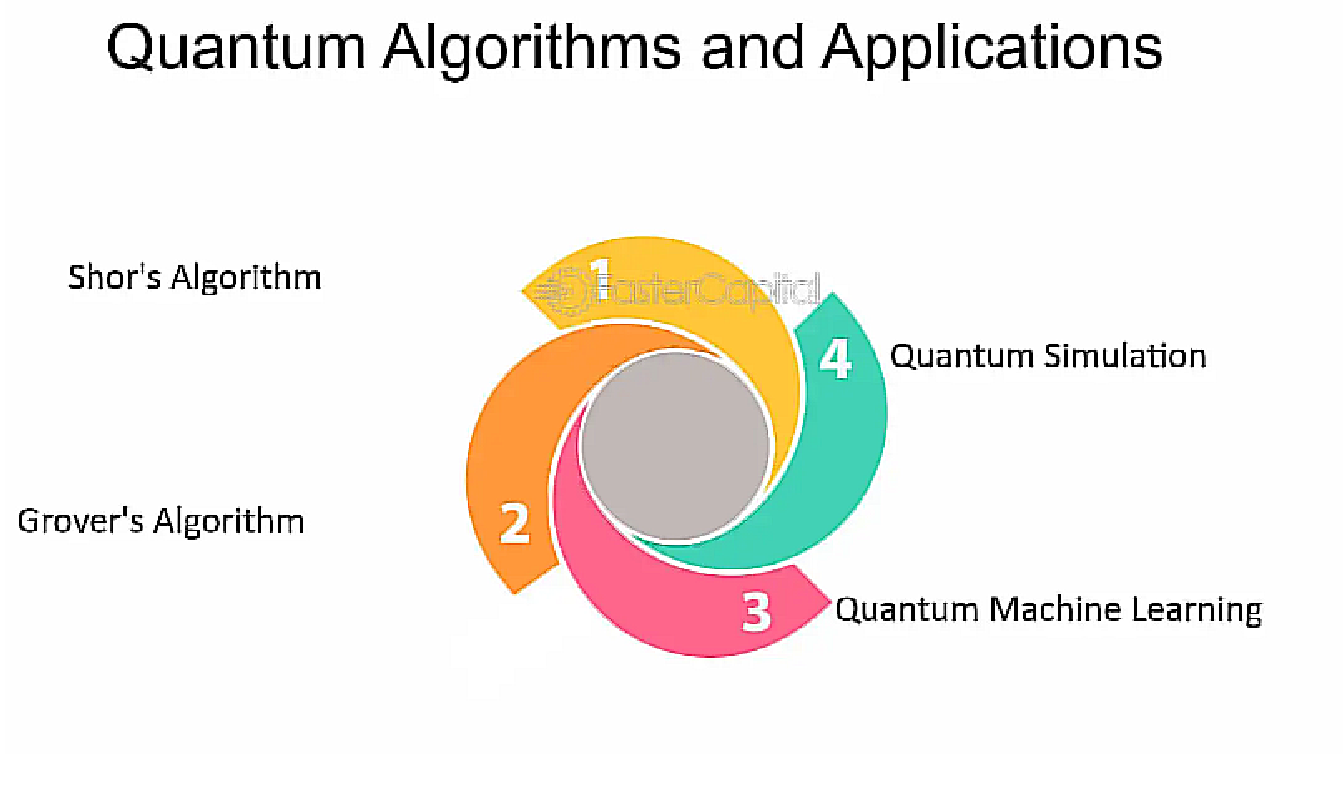


As we all know, **Quantum Computing** is an emerging tech field integrated from both *Physics and Computer Science.* Even though it is derived from **Computer Science**, it was created to be much more efficient, the new algorithms were invented for people to use.
These algorithms are known as Quantum Algorithms. A few examples of Quantum Algorithms would be **Shor's Algorithm and Grover's Algorithm**, both of which we studied during the course but today I will talk about *QPE(Quantum Phase Estimation).*

References Used:

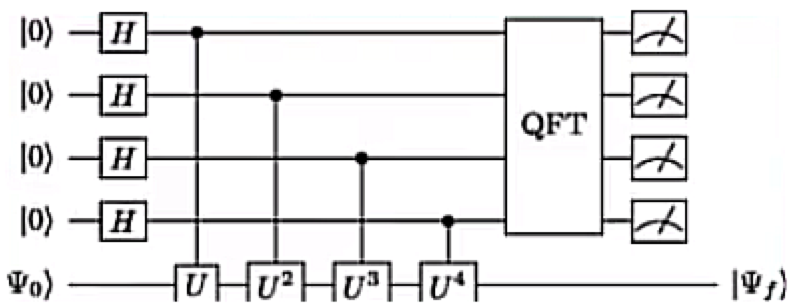
- **Wikipedia** for details
- *Google* for images

What is QPE?

QPE, Pioneered by *Alexei Kitaev in 1955*, is an algorithm used as a **sub-routine**(*smaller named sections of code* that are written within a **larger program** close program Sequences of instructions for a computer). In many other algorithms like **Shor's factoring algorithm** and **Quantum Amplitude Esimation algorithms.**

So now we know that **what it is.** But what does it *exactly do?* And how exactly *does it work?*

This Algorithm was a *pivotal* designed to **estimate the phase** associated with an **eigenvalue** of a given **unitary operator** possess unit modulus, *their essence is defined by their phase*. Consequently, the algorithm can be **effectively framed** as retrieving either the **phase** or the ***eigenvalue itself.*** All of this can be explained by the image below:



Workings of QPE

Here's a ***breakdown*** of what it exactly does:

Input:

The *algorithm* takes **2 key components** as inputs:

- A **quantum circuit** representing a *unitary operator* whose **eigenvalues** we want to analyze.
- An **eigenvector** which servers as the **Input state**.

Encoding the EigenValue:

$$Ax - \lambda x = 0$$
$$(A - \lambda I)x = 0$$

QPE works by encoding the **phase** corresponding to the *eigenvalue of interest* onto a set of *ancilla qubits.* These **ancilla Qubits** are then further prepared in a **superposition state.**

EigenValues are the special set of *scalar values* that is associated with the **rest of the linear equations**, original is used to study **principal axes** of the *rotational motion of rigid bodies*, **eigenvalues and eigenvectors** have a wide range of **applications**, for example **instability analysis**, **atomic orbitals**, **facial recognition** and **matrix diagonalization.**

ancilla bits are *extra bits* being used to implement **irreversible logical operations.** In classical computation, any memory bit can be **turned on or off at will**, requiring no prior knowledge or **extra complexity.**

Quantum Circuit:

Quantum Phase Estimation employs a **quantum circuit** compose of *controlled unitary operations* to entangle the state of the **ancilla qubits** with the *phase information* from the *eigenvector.* This process effectively *encodes* the phase onto the **ancilla qubits**.

Phase estimation:

The algorithm then performs an *inverse quantum Fourier transorm*(QFT is a linear transformation) on the **ancilla qubits**. This transforms the **phase information** encoded in the **ancilla qubits** into a **classical register**.

Measurement:

Finally the **classical register** is measured, yielding an *approximation* for the phase *corresponding to the eigenvalue* of the **unitary operator**.

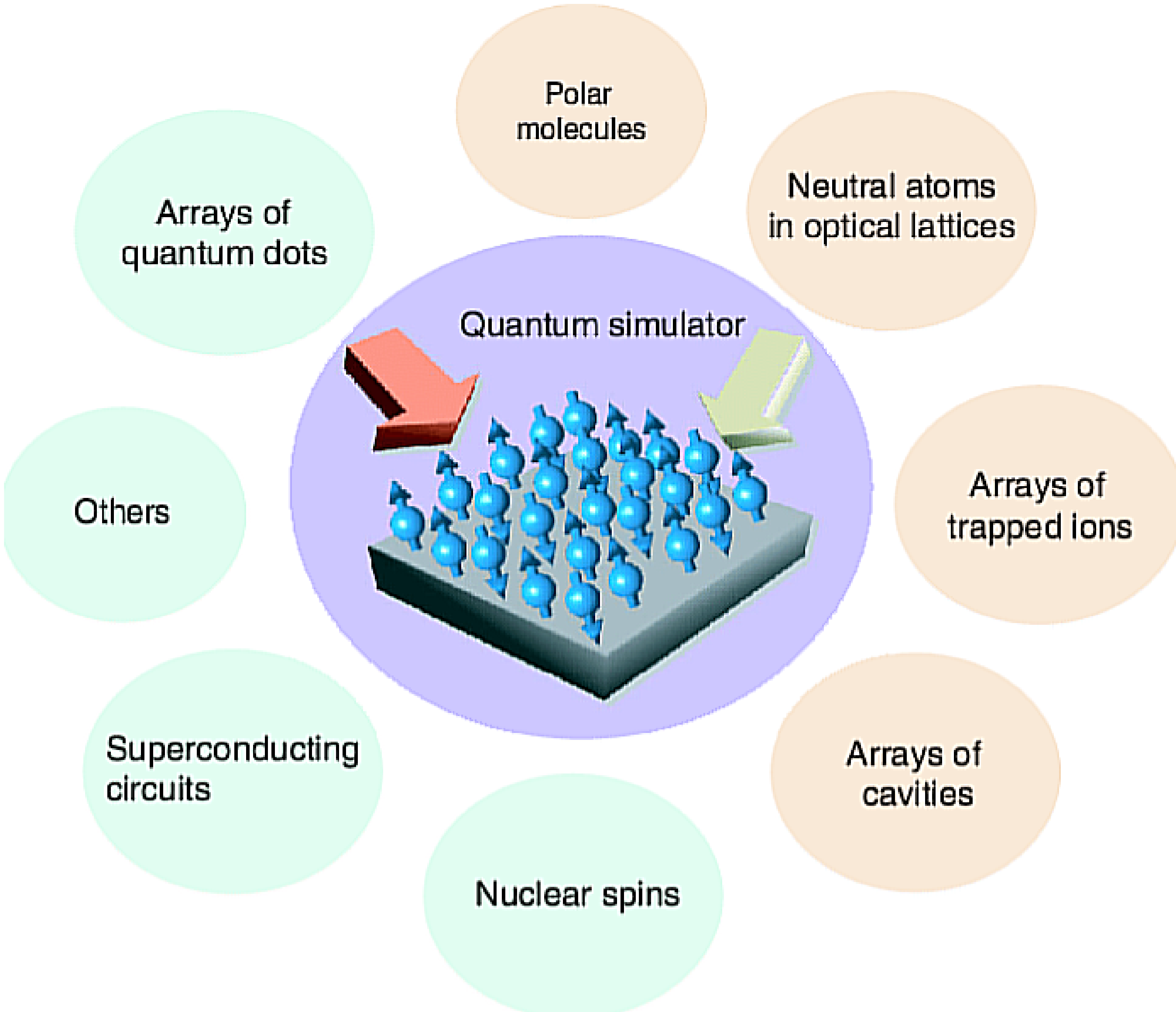
Output:

The *measurement outcode* represents the **estimated phase**. By analyzing the phase, one can infer the properties of the **eigenvalues** of the *unitary operator*, which is valueable for various **quantum algorithms and simulations**.

Uses of QPE in now and in the future

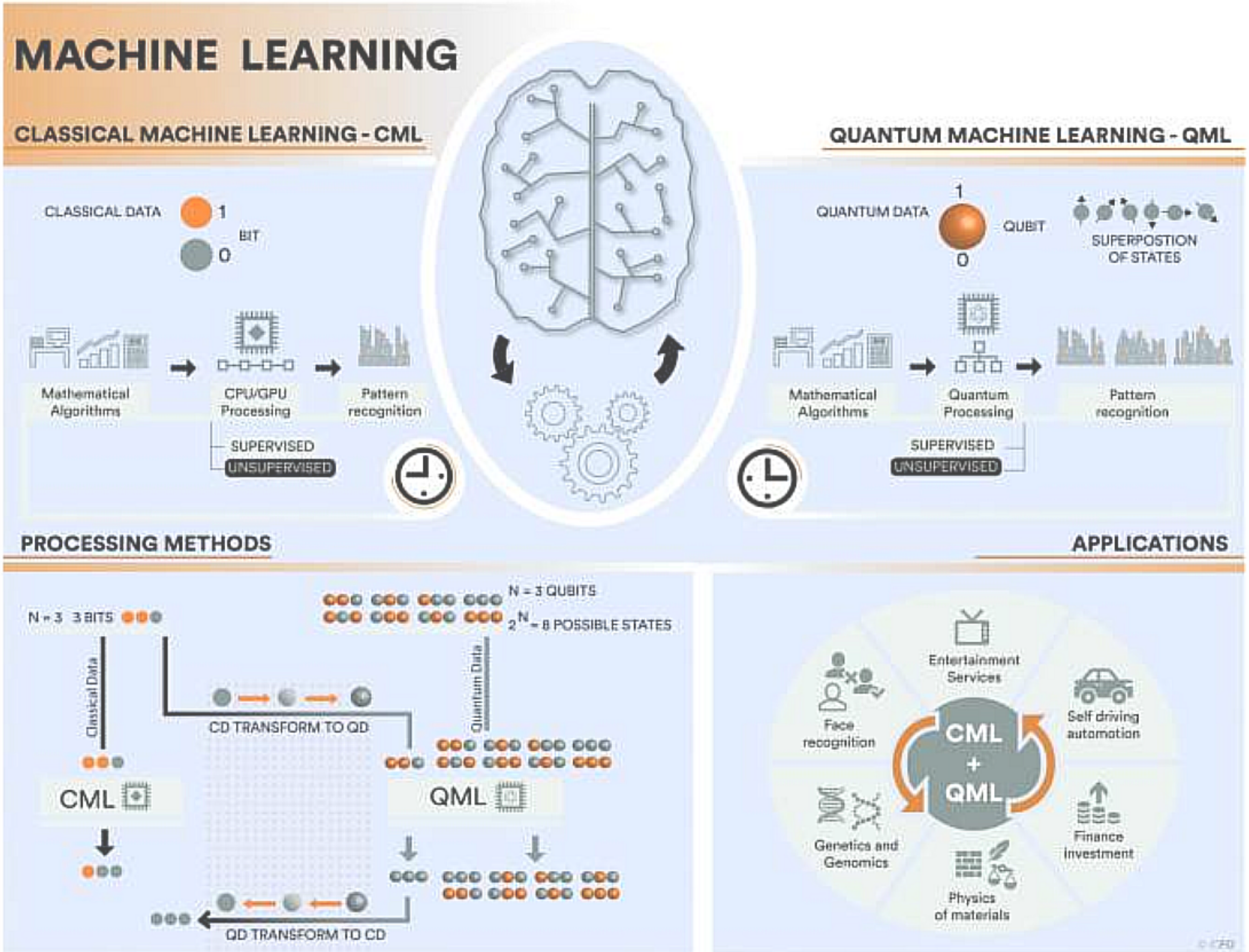
The QPE algorithm holds **significant promises** for various applications in future computing and **related fields**. Some potential uses includes things such as:

Quantum Simulation



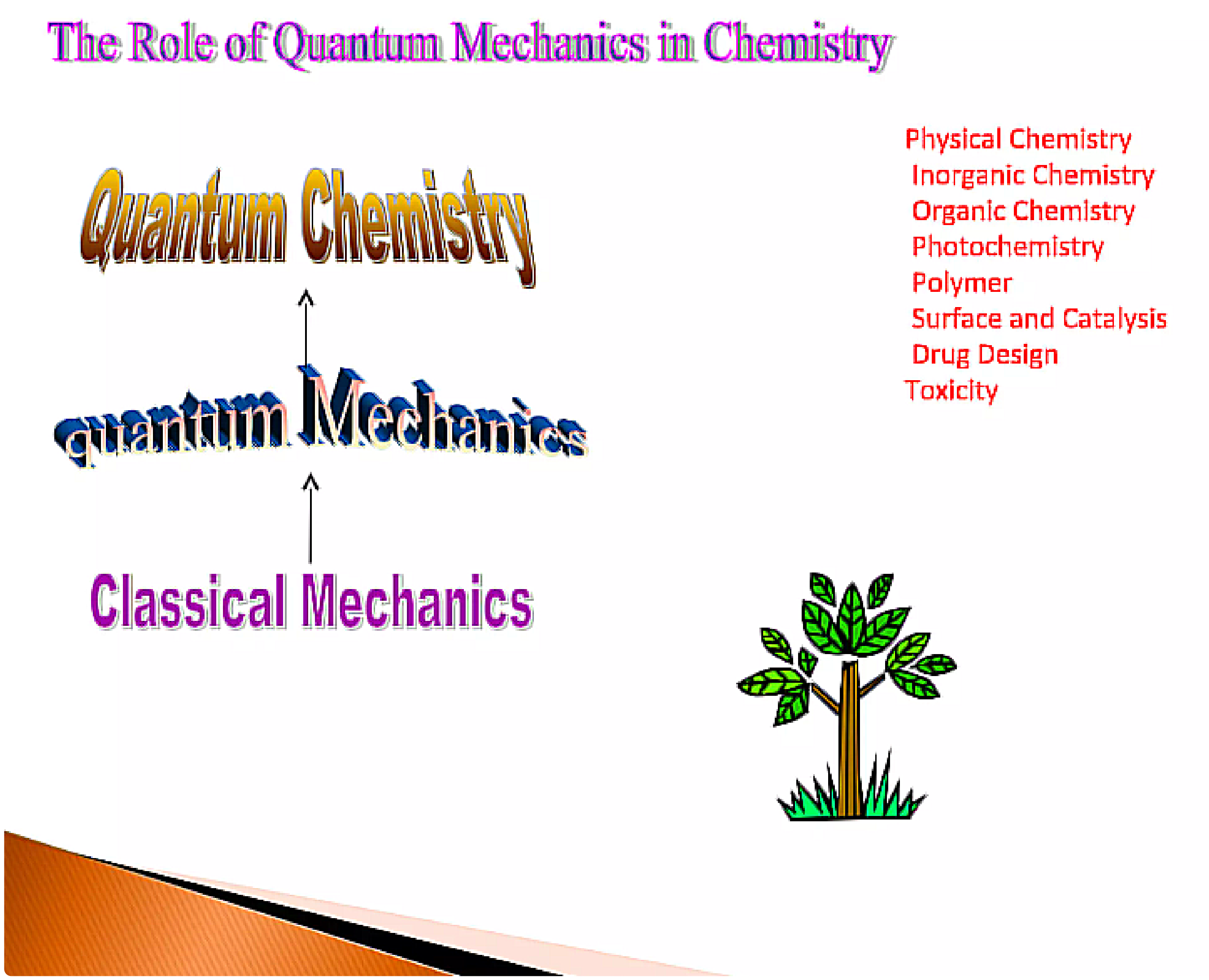
Allows **researchers** to estimate properties of **quantum systems.**

Quantum Machine Learning



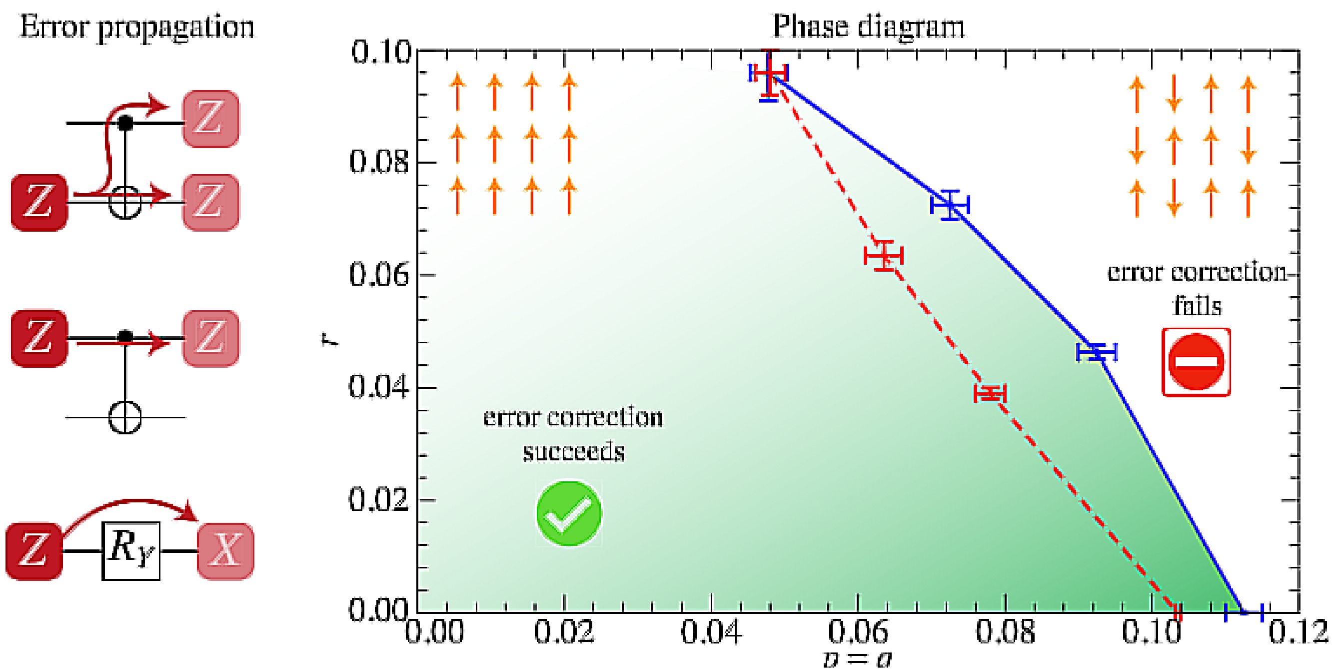
Enables **efficient representation** and **manipulation** of *quantum data*, leading to advancements in **quantum enhanced** machine learning models.

Quantum Chemistry



Enables the calculation of **molecular energies** and **electronic structures** with high precision

Quantum Error Correction

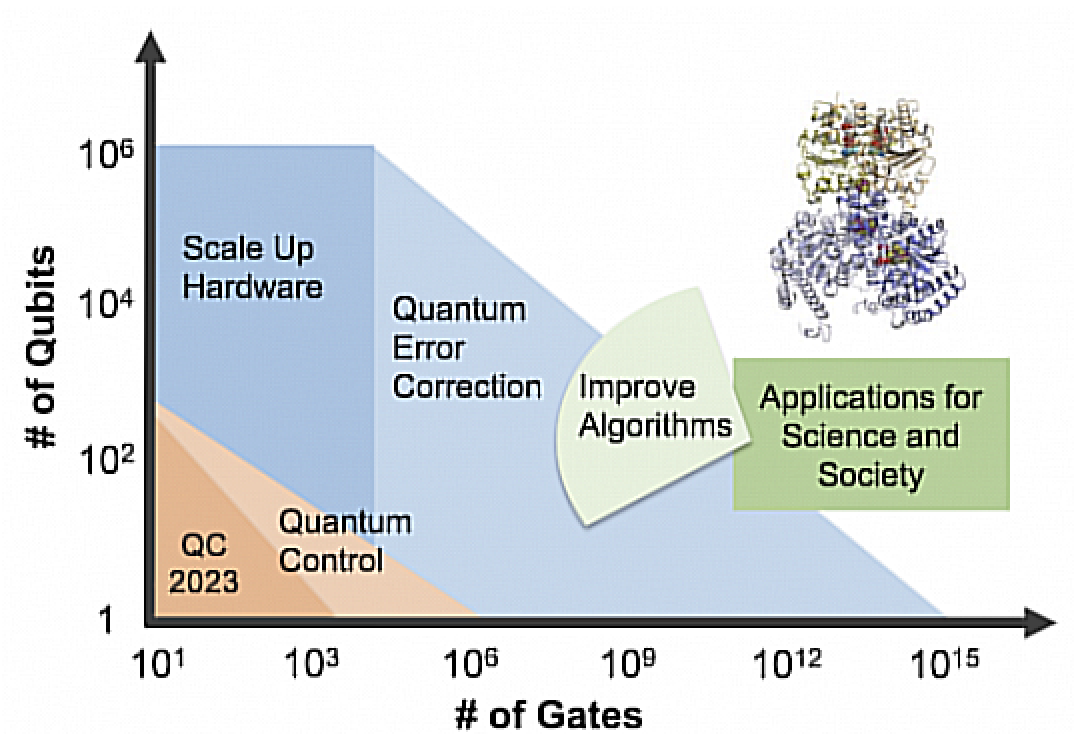


QPE plauses a crucial role in fault tolerant *quantum computation* and *error correction*

Quantum Cryptography

It has porential applications in **quantum cryptography**, particularly in protocols such as ***Quantum key distribution***

Quantum Algorithms



QPE servers as a **key subroutine** in various **quantum algorithms**, including *Shor's algorithm*

Summary:

The **Quantum Phase Estimation algorithm** essentially provides a **quantum means of estimating** the *phase* associated with the **eigenvalues** of a *given unitary operator*, which is **crucial** for understanding the *behaviour of quantum systems* and designing **quantum algorithms** and more as shown before.